Two angles are ________________ when they have the same initial and terminal sides. To find an angle that is coterminal to a given angle θ add or subtract 2π (one revolution). A given angle θ has ________________ many coterminal angles.

Coterminal Angles

If θ is the measure of an angle in radians, then all angles measuring $\theta + 2n\pi$, where *n* is an integer, are coterminal with θ.

Example 1: For each problem draw the angles/ coterminal relationship.

a. For the positive angle $\frac{7\pi}{4}$, subtract 2π to obtain a coterminal angle.

b. For the positive angle $\frac{5\pi}{6}$, subtract 2π to obtain a coterminal angle.

c. For the negative angle $-\frac{3\pi}{4}$, add 2π to find a coterminal angle.

Example 2: Find and draw a positive and negative coterminal angle with the given angle

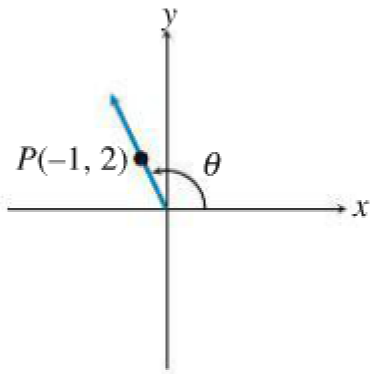
a. 40°

b. -120°

c. $\frac{2\pi}{3}$ radians

In Exercises 3–6, evaluate the six trigonometric functions of the angle θ.

3.



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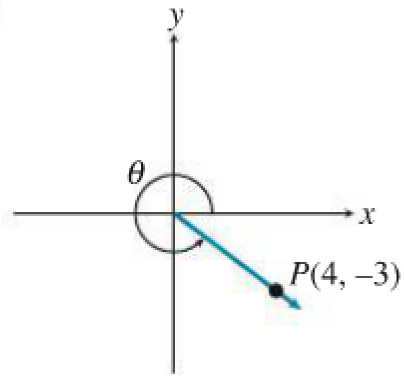

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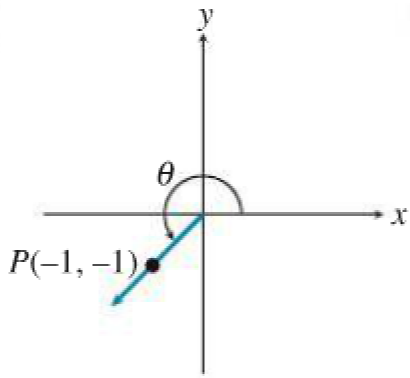

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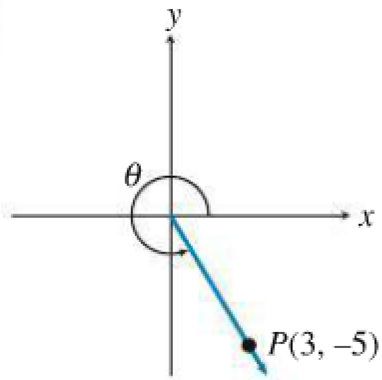

In Exercises 7–12, point P is on the terminal side of angle θ. Evaluate the six trigonometric functions for θ. If the function is undefined, write "undefined."

7. $P(3, 4)$

8. $P(-4, -6)$

9. $P(0, 5)$

10. $P(-3, 0)$

11. $P(5, -2)$

12. $P(22, -22)$

In Exercises 13–16, state the sign $(+ \text{ or } -)$ of **(a)** $\sin t$, **(b)** $\cos t$, and **(c)** $\tan t$ for values of t in the interval given.

13. $\left(0, \frac{\pi}{2}\right)$

14. $\left(\frac{\pi}{2}, \pi\right)$

15. $\left(\pi, \frac{3\pi}{2}\right)$

16. $\left(\frac{3\pi}{2}, 2\pi\right)$

In Exercises 25–36, evaluate without using a calculator by using ratios in a reference triangle.

25. $\cos 120°$

26. $\tan 300°$

27. $\sec \frac{\pi}{3}$

28. $\csc \frac{3\pi}{4}$

29. $\sin \frac{13\pi}{6}$

30. $\cos \frac{7\pi}{3}$

31. $\tan -\frac{15\pi}{4}$

32. $\cot \frac{13\pi}{4}$

33. $\cos \frac{23\pi}{6}$

34. $\cos \frac{17\pi}{4}$

35. $\sin \frac{11\pi}{3}$

36. $\cot \frac{19\pi}{6}$

In Exercises 37–42, find **(a)** $\sin \theta$, **(b)** $\cos \theta$, and **(c)** $\tan \theta$ for the given quadrantal angle. If the value is undefined, write "undefined."

37. $-450°$

38. $-270°$

39. 7π

40. $\frac{11\pi}{2}$

41. $-\frac{7\pi}{2}$

42. -4π

In Exercises 43–48, evaluate without using a calculator.

43. Find $\sin \theta$ and $\tan \theta$ if $\cos \theta = \frac{2}{3}$ and $\cot \theta > 0$.

44. Find $\cos \theta$ and $\cot \theta$ if $\sin \theta = \frac{1}{4}$ and $\tan \theta < 0$.

45. Find $\tan \theta$ and $\sec \theta$ if $\sin \theta = -\frac{2}{5}$ and $\cos \theta > 0$.

46. Find $\sin \theta$ and $\cos \theta$ if $\cot \theta = \frac{3}{7}$ and $\sec \theta < 0$.

47. Find $\sec \theta$ and $\csc \theta$ if $\cot \theta = -\frac{4}{3}$ and $\cos \theta < 0$.

48. Find $\csc \theta$ and $\cot \theta$ if $\tan \theta = -\frac{4}{3}$ and $\sin \theta > 0$.

We are now ready to state the formal definition.

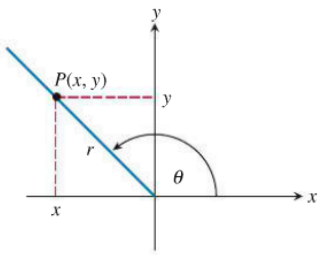

FIGURE 4.25 Defining the six trig functions of θ.

DEFINITION Trigonometric Functions of Any Angle

Let θ be any angle in standard position and let $P(x, y)$ be any point on the terminal side of the angle (except the origin). Let r denote the distance from $P(x, y)$ to the origin, i.e., let $r = \sqrt{x^2 + y^2}$. (See Figure 4.25.) Then

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}\ (y \neq 0)$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}\ (x \neq 0)$$

$$\tan\theta = \frac{y}{x}\ (x \neq 0) \qquad \cot\theta = \frac{x}{y}\ (y \neq 0)$$